# Set Theory

* A **set** is a loosely defined collection of items called **elements**.
* Sets are completely determined by their elements, i.e. two sets with exactly the same elements are the same set.
* The order in which elements are listed is irrelevant, and elements may be listed more than once without changing the set.
* Examples:

The collection of all people in this room is a set.

The collection of your favourite songs is a set.

The collection of all real numbers is a set.

* Sets come from a **universe** of elements .
* For example, the set of even numbers comes from the universe .
* Sets can be contained in other sets and can be finite or infinite.
* Some important sets of numbers are:
* (NATURAL)
* (INTEGER)
* (RATIONAL)
* (RATIONAL AND IRRATIONAL)
* A set can be defined by a property of elements of a bigger set.
* Given a set S, define a set T by:

All elements of S that satisfy p.

Example:

The set is the set of all real numbers between -2 and 5, not including -2. This set is an internal, which can be denoted as .

Exercise:

The set can be rewritten how?



Exercise:



The set can be rewritten how?



* The **empty set** is the set with no elements, denoted by .
* It can be represented in different ways:
* A set is **finite** if such that there is a one-to-one correspondence with the set .
* For a set of this size, we write and say that has **cardinality**
* NOTE:
* A set that is not finite is said to be **infinite**.

## Subsets

#### Definition:

* Let and be sets.
* We say is a subset of , written , IFF every element of is also an element of .

|  |
| --- |
| Definition – Subsets: |
|  |

## Supersets

#### Definition:

* If is a subset of .
* Then is called a superset of .
* We also say that is contained in , and that contains .
* If at least one element of is not in , then is not a subset of .

|  |
| --- |
| Definition – Supersets: |
|  |

Exercise:

Decide true or false.



## Proper Subsets

#### Definition:

* A subset is **proper** if .
* We write
* For example, and , but and , so actually .

|  |
| --- |
| Definition – Proper Subsets: |
|  |

Exercise:

Order the sets in terms of subsets. Are any of these proper subsets?



Exercise:

True or false? Let .



#### Definition:

* Let and be sets.
* We say **equals** , written , if and only if, contains and contains .

|  |
| --- |
| Definition – Set Equality |
|  |

* To prove two sets are equal, prove the two contentions, and .

Exercise:

Prove that and are equal.

Let . Then for some .

.

Let , so is even.

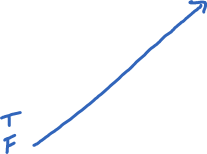
Suppose that is odd, for some .

Then is a

Hench, is even and .

Exercise:

Define:



1. Is ?



1. Is ?



1. Is ?



### Operations on Sets

* Let be subsets pf a universe /

1. The **union** of and ,written , is the set of all elements that are in or in .
2. The **intersection** of and , written , is the set of all elements that are in and in .
3. The **complement** of , written or , is the set of all elements that are not in .
4. The **difference** of minus , written , is the set of all elements that are in and not in .

## Power Set

#### Definition:

* The **power set** of a universe , denoted by , is the set of all subsets of .

Exercise:

Let .

* If , then
* The operations of set theory are equivalent to their counterpart connectives of logic, as follows:

|  |  |  |
| --- | --- | --- |
| Set Operation | Name | Connective |
|  | **Complement** |  |
|  | **Union** |  |
|  | **Intersection** |  |
|  | **Subset** |  |
|  | **Equality** |  |

Exercise:

Let . Write down for the following.









Exercise:

Let . Write down for the following, and .




2. is the set of even integers, is the set of odd integers.
3. and

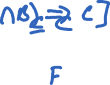
Exercise:

Prove or disprove: .



Exercise:

Prove or disprove:



Exercise:

Let .










## Disjoints

#### Definition:

* The sets and are **disjoints** if .

Exercise:

Let . Write down some sets that are disjoint to the following.





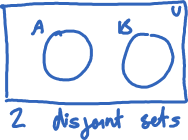
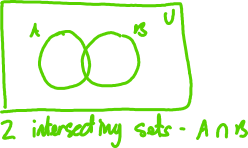
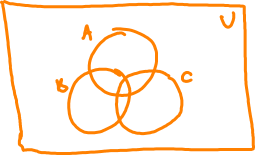



#### Definition (Addition Rule):

* Let be finite, disjoint sets.
* Then is finite and .

## Venn Diagrams

* If we represent sets as regions in the plane, then the relationships among sets can be represented by drawing called **Venn diagrams**.



## Algebra on Sets

* There are many rules that govern set theory and the relationships among sets.
* All the following statements can be proved using the definitions we have seen so far.

### Theorem:

* Let be a set, and be element of . Then…

4. **Commutative Laws:**
5. **Associative Laws:**
6. **Distributive Laws:**
7. **De Morgan’s Laws:**

Exercise:

Prove (7) .



Exercise:

Prove (2) .



Exercise:

Prove that the difference operator is not commutative.



## Pairwise Disjoint

#### Definition:

* The sets are **pairwise disjoint** if for all .

### Theorem (Extension of Addition Rule):

* Let be finite, pairwise disjoint sets.
* Then is finite and .